The measurements on the interaction between molten cobalt and VK8 alloy specimens (7.6 wt. % Co 92.4 wt. % WC) were made by the method described in [2]; the times of contact between the VK8 specimens and the molten cobalt saturated with tungsten and carbon were 180, 300, 480, 720, 1080, and 1380 sec. The cobalt distribution was examined by x-ray spectral microanalysis; the following results were obtained for VK8 alloy: $a = 0.6$; $C = 2.5 \cdot 10^{-6}$ m^2/N; $D = 1.86 \cdot 10^7$ $N \cdot sec^{b/2}/m^{2+b}$; $b = 1.44$. The measurements agree well with the calculations (Fig. 2).

Therefore, (5) can be used in quantitative determination of the amount of bonding metal in a sintered material and in analyzing the kinetic characteristics.

Also, (5) can be used in research on other processes described by differential equations of parabolic type, e.g., thermal conduction, diffusion, gas infiltration, and motion of ions in an electric field. However, (5) can be used only if a negative-exponential relationship applies for the extensive quantities (heat, mass, and volume) in terms of the intensive quantities that provide the driving force (temperature, chemical potential, and pressure), i.e., if the conditions represented by (3) are met.

NOTATION

Π, migration pressure; r, particle size; δ, amount of liquid phase as a volume fraction; k_m, migration coefficient; k_p, permeability coefficient; ϑ, viscosity; x, coordinate; τ, time; A, a, E, G, C, D, coefficients; b, m exponents.

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X-RAY STUDY OF DRYING IN A CAPILLARY POROUS BODY

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UDC 66.047.9: 539.26

Nonisothermal transport of moisture in a capillary porous body was studied by an x-ray method. Radiometry of the radiation and precision temperature control of the drying sample are discussed.

Absorption measurements of radiation passing through a sample should be recognized as a most effective method for studying the motion of moisture in a capillary porous body during drying. Gamma radiation from radioactive isotopes is being used successfully for this purpose [1, 7, 8]. Because of the high energy of the radiation quanta, however, the mass absorption coefficient μ is extremely small and the beam path in a sample is large (tens of centimeters); this creates experimental difficulties connected with temperature control and equalization of air flow above the drying surface in a constant-temperature device. Therefore, bremsstrahlung x-rays were used in the present work [2]. The presence of absorbing walls of the sample holder, of sample material, of the covering of the radiation detector, and of the beryllium window of the x-ray tube makes it

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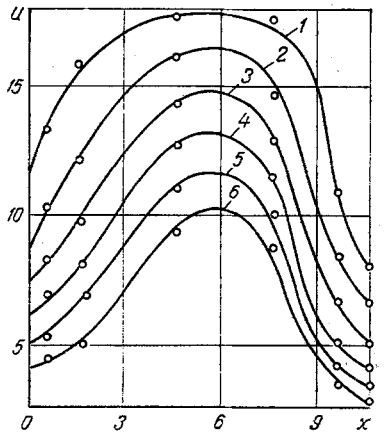

Fig. 1. Dependence of moisture content u, %, on coordinate x, mm, at various times (min): 1) 70; 2) 80; 3) 90; 4) 100; 5) 110; 6) 120.

possible to realize an exponential law for radiation attenuation by the water contained in a sample. Experimental calibration was performed with a set of plane-parallel Plexiglas containers of known thickness set up in the path of the x-ray beam with a dry sample; it gave an attenuation coefficient $\mu = 0.44$ cm^2/g for water with the tube voltage stabilized at 27 kV.

We consider a linear theory for asymmetric drying of a layer sealed against moisture from below and drying upward at a constant rate. Experiment shows that with good temperature control of the bottom of the sample holder, quartz sand dries with a temperature gradient which is constant in space and time and in accordance with theory [3] the process is described by the following equations:

$$\frac{\partial u(x,\ t)}{\partial t} = a_m \frac{\partial^2 u(x,\ t)}{\partial x^2}\ , \tag{1}$$

$$u(x,\ 0) = u_0 = \text{const}, \tag{2}$$

$$\frac{\partial u(0,\ t)}{\partial x} + \delta\tau = 0, \tag{3}$$

$$\frac{\partial u(L,\ t)}{\partial x} + \delta\tau = -\frac{G}{a_m \rho_0}\ . \tag{4}$$

Equation (1) is the equation of mass transfer for $\tau = \partial\theta/\partial x = \text{const}$; Eq. (2) is the initial condition for uniform distribution of moisture; Eqs. (3) and (4) are, respectively, the boundary conditions at the moisture-sealed bottom of the sample holder and at the drying surface.

Performing a Laplace transform of Eqs. (1)-(4), we find a transform similar to that in [4] (Chap. V, Sec. 2). Its inverse has the form

$$u = u_0 + \delta\tau\left[\frac{a_m t}{L} - \frac{L^2 - 3(L-x)^2}{6L} + L\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2}{n^2\pi^2}\cos n\pi\left(1-\frac{x}{L}\right)\exp\left(n^2\pi^2\frac{a_m t}{L^2}\right)\right].$$

TABLE 1. Signs of Derivatives in Various Sections of Moisture Distribution Curve

Method	Derivative	Section			
		1	2	3	4
Simplified analytic solution	$\partial u/\partial x$	+	+	−	−
	$\partial^2 u/\partial x^2$	−	−	−	−
Numerical differentation	$\partial u/\partial x$	+	+	−	−
	$\partial^2 u/\partial x^2$	+	−	−	+

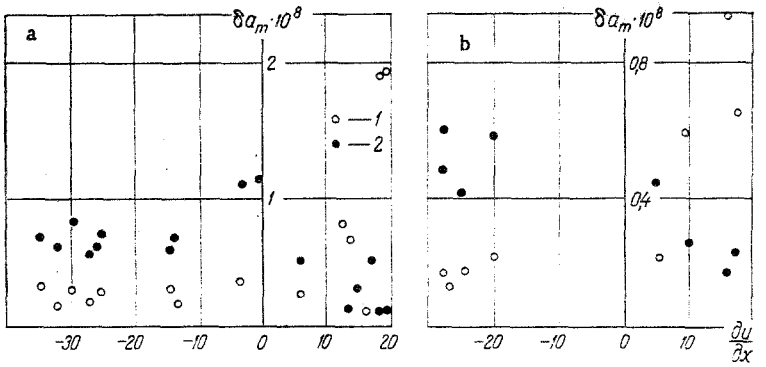

Fig. 2. Dependence of a_m, m^2/sec (1) and δ, $°K^{-1}$ (2) on $\partial u/\partial x$, m^{-1}. Quartz sand fraction, 0.25-0.5 mm: a) bottom temperature 32°C, motor voltage 12.2 V; b) bottom temperature 37°C, motor voltage 4.8 V.

$$-\left(\delta\tau+\frac{G}{a_m\rho_0}\right)\left[\frac{a_m t}{L}-\frac{L^2-3x^2}{6L}\right.$$

$$\left.+L\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2}{n^2\pi^2}\cos n\pi\frac{x}{L}\exp\left(-n^2\pi^2\frac{a_m t}{L^2}\right)\right]$$

or after elimination of the sums which go to zero,

$$u=u_0+\delta\tau\ \frac{L-2x}{2}-\frac{G}{a_m\rho_0}\left[(L^2-3x^2)\ \frac{1}{6L}-\frac{a_m t}{L}\right]. \tag{5}$$

The solution (5) gives a parabolic distribution of moisture content along the coordinate with a linear drop in time, i. e., it is somewhat analogous to a regular thermal mode of the second kind.

For the purpose of experimental investigation of drying of a capillary porous body (sand with grain sizes 0.25-0.5 mm cleaned by the standard method [5]), laboratory equipment was built the main portion of which was a vertically displaceable constant-temperature device; a sample holder with horizontal dimensions of 21×21 mm^2 was installed inside it. On its lateral surface, the sample holder was lined with foam plastic 10 mm thick. The vertical dimension of the sample holder was 15 mm. The displacement of the constant-temperature device along the vertical was measured with a cathetometer. The sample was irradiated horizontally by the beam from the x-ray tube. Fans were installed inside the constant-temperature device and the speed of the fan motors was easily changed by means of a rheostat.

An automatic control system was used to regulate sample temperature. The temperature sensor in this system was a thermocouple with its hot junction attached to the outer surface of the bottom of the sample holder (a brass plate 2 mm thick) and its cold junction in a Dewar flask located outside the constant-temperature device. The thermocouple was connected to the input of a single-point recording potentiometer (type ÉPP-09) through a balancing voltage divider and photoelectric amplifier (type F-117) with a voltage amplification factor up to 100. By means of a flag attached to the carriage, this potentiometer controlled a light beam incident on a photoresistor connected to a phase shifter and, by means of a thyratron unit, changed the operating current of the heater in the constant-temperature device, keeping the temperature of the bottom of the sample holder constant to within ± 0.03°K. The experimental distribution of moisture content over the height of the sample holder was obtained from analysis of the radiometric measurements. The measurements of moisture content refer to 5-7 layers located at different distances from the bottom of the sample holder and irradiated in turn. A description of the x-ray equipment for irradiation and monitoring is found in [2] and the appropriate formulas, in [7] and [8]. The background signal recorded by the detector was taken into consideration.

A scheme for thermometric measurements, which is not shown in the figures, recorded the results of temperature measurements by a second multipoint recording potentiometer through a second photoelectric amplifier and balancing voltage divider. The hot junctions of copper—Constantan thermocouples provided for these measurements were arranged along the height of the sample holder at distances x = 0, 3, 6, 9, and 12 mm from the inside bottom surface.

The diameter of the wires from which the thermocouples were made did not exceed 0.1 mm. The cold junctions of all thermocouples were in a Dewar flask. The thermocouple switch was a double-pole switch breaking two copper conductors to avoid harmful galvanic connections. The sensitivity of the temperature recordings could be changed. In most measurements it corresponded to 10°K full scale. The total temperature drop between the bottom and the surface of the sample was 1-4°K. Containers of calcium chloride were installed in order to standardize experimental conditions with respect to the air humidity inside the constant-temperature device.

Experiment showed that the solution (5) only agrees qualitatively with experiment in the presence of a maximum moisture content between the bottom of the sample holder and the drying surface. Therefore, an attempt was made at direct analysis of the experimental data.

By computer, equations of Lagrangian interpolation polynomials were found from the experimental results with u(x) being the polynomial for each interpolation curve from the coefficients of which the derivatives $\partial u/\partial x$ and the integrals $\int_0^x u(x)dx$ were determined. Each group of derivatives and integrals was calculated for a single time with a coordinate spacing of 1 mm. Differentiation of the integral curves with respect to time in order to determine moisture flow densities was performed graphically in order to avoid errors which might possibly result from a second analytic approximation. This stage of the work was relatively simple because of the slight curvature of the integral curves.

Keeping in mind the equation of mass transport [3] for τ = const, we obtain for two neighboring points

$$G_1 = -a_m\rho_0\left(\frac{\partial u_1}{\partial x}\right) - a_m\rho_0\delta\tau,$$
$$G_2 = -a_m\rho_0\left(\frac{\partial u_2}{\partial x}\right) - a_m\rho_0\delta\tau, \tag{6}$$

$$a_m = \frac{G_2 - G_1}{\rho_0\left(\frac{\partial u_1}{\partial x} - \frac{\partial u_2}{\partial x}\right)}, \quad \delta = -\frac{\frac{G_1}{a_m\rho_0} + \frac{\partial u_1}{\partial x}}{\tau}. \tag{7}$$

If we divide the distribution curve constructed from the linear solution (5) and the curves reflecting the results of numerical analysis into four successive sections along the coordinate, we then obtain in these sections derivatives with signs which alternate as shown in Table 1.

Figure 1 shows an example of moisture distribution curves obtained by the x-ray method. Results from numerical solution of Eqs. (7) are shown in Figs. 2a, b. Of course, operations based on graphical analysis of primary materials are of limited accuracy, but consideration of them leads to the following conclusions.

1. Distribution curves for moisture content over the height of the sample holder for moisture contents exceeding the meniscus content determined by a drying thermogram (4.5%) have a clearly expressed maximum.

2. For negative and positive values of the argument $\partial u/\partial x$ largest in modulus, the function a_m has negative values (not shown in Fig. 2). It is easy to confirm that this situation occurs when $\partial^2 u/\partial x^2 > 0$.

This kind of dependence for u(x) is encountered in the literature [6] but as far as we know is nowhere described in detail.

3. At comparatively intense drying (bottom temperature 41°C, motor voltage 12.2 V), the dependence of the coefficients a_m and δ on process parameters as shown in Fig. 2 is difficult to represent despite the smoothness of the moisture distribution curves. It is apparent that the previously used method for data analysis from two points does not provide sufficient accuracy for so intensely varying gradients of the moisture content along the coordinate.

4. The position of the maximum moisture content in the sample holder is distinctively different in different experiments; it shifts toward the drying surface as the temperature increases. In our experiments, the coordinates of the maximum moisture content were 0.56, 0.66, and 1.1 cm for sample-holder bottom temperatures of 32, 37, and 41°C, respectively.

NOTATION

u(x), distribution curve for moisture content over sample height; x, coordinate measured along height of sample with x = 0 at bottom of sample holder; ρ_0, sample density; G, G_1, G_2, moisture flow densities; a_m, moisture diffusion coefficient; δ, moisture thermodiffusion coefficient; τ, absolute value of temperature gradient; t, time; Θ, temperature, L, sample height.

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OPTIMUM CONTROL OF PROCESSES OF HEAT AND MASS TRANSFER DURING DRYING

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UDC 66.074.31

The problem of the optimum control of processes of conjugate heat and mass transfer during drying is solved for the case when the controlling parameter is the temperature of the drying agent and the quality criterion is the heat expenditure.

Despite the fact that the structure of the system of differential equations describing the processes of heat and mass transfer during drying is well known [1], the use of mathematical methods of optimization of drying processes is complicated by a number of factors, among which one must include the absence of analytical expressions for the coefficients of the equations which are functions of the parameters of the drying process and the absence of sufficiently reliable methods of optimization. Even if analytical expressions for the coefficients of the system are known, their substitution into the equations of the system so complicates the latter that one is often not able to use exact methods of optimization. The replacement of the variable coefficients of the system by constants reduces the accuracy of the mathematical description of the process and in a number of cases leads to the loss of the connection between the controlled and the controlling parameters, which eliminates the possibility of optimization using the given model. In this connection the development of approximate methods of optimization for a system of equations with variable coefficients is an urgent task.

It is known that the drying process in the general case is described by a system of differential equations of conjugate heat and mass transfer proposed by A. V. Lykov [2] in which the coefficients to the partial derivatives are combinations of the thermophysical and thermodynamic characteristics of the material being dried. For each concrete process they are different functions of the principal drying parameters: the temperature and moisture content.

By replacing these functions by constants we obtain a new simplified system of equations with constant coefficients, the solution of which is considerably simplified.

We will distinguish a class of these simplified systems of equations, each of which has a solution, and for one of the systems we will find an approximate method of solution of the problem of the optimum control of drying processes.

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